ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
CAMPUS DI FORLÌ

91258 / B0385
Natural Language Processing

Lesson 5. Naïve Bayes' Classifier)

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13/10/2025

Previously

- One rule-based system for sentiment analysis
- Brief introduction to ML
- Foreword to Naïve Bayes

Naïve Bayes

Naïve Bayes

1. Introduced in the IR community by Maron (1961)
2. First machine learning approach
3. It is a supervised model
4. It applies Bayes' theorem with strong (naïve) independence assumptions between the features:
   - they are independent
   - they contribute "the same"

Naïve Bayes
A conditional probability model

Given an instance represented by a vector

$$\mathbf{x} = (x_1, \ldots, x_n) \tag{1}$$

representing n independent features $x_1, x_2, x_3, \ldots, x_{n-2}, x_{n-1}, x_n$

n could be $|V|$ (the size of the vocabulary)[1]

The model assigns to instance $\mathbf{x}$ the probability

$$p(C_k \mid \mathbf{x}) = p(C_k \mid x_1, \ldots, x_n) \tag{2}$$

for each of the k possible outcomes C_k

where $C_k = \{c_1, \ldots, c_k\}$

From https://en.wikipedia.org/wiki/Naive_Bayes_classifier

[1] $|\mathbf{x}|$ is the cardinality of $\mathbf{x}$

Naïve Bayes'
Using Bayes' Theorem

The conditional probability $p(C_k \mid x_1, \ldots, x_n)$ can be decomposed as

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\ p(\mathbf{x} \mid C_k)}{p(\mathbf{x})} \tag{3}$$

Which can be read as

$$\text{posterior} = \frac{\text{prior} \times \text{likelihood}}{\text{evidence}}$$

But $p(x)$ does not depend on the class (since it is constant):

$$p(C_k \mid \mathbf{x}) \sim p(C_k)\ p(\mathbf{x} \mid C_k) \tag{4}$$

From https://en.wikipedia.org/wiki/Naive_Bayes_classifier

Naïve Bayes
Going deeper (assuming a binary classifier)

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{5}$$

$$\text{posterior probability} = \frac{\text{class prior probability} \times \text{likelihood}}{\text{predictor prior probability}}$$

$p(C \mid \mathbf{x})$ Posterior probability of the class given the input[2]

```
if p > 0.5:
    class = positive
else:
    class = negative
```

[2] [illegible]

Naïve Bayes
Going deeper (assuming a binary classifier)

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{6}$$

$$\text{posterior probability} = \frac{\text{class prior probability} \times \text{likelihood}}{\text{predictor prior probability}}$$

$p(C)$ Class prior probability
How many positive instances I have seen (during training)

Naïve Bayes

Going deeper (assuming a binary classifier)

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \quad (7)$$

$$\text{posterior probability} = \frac{\text{class prior probability} \times \text{likelihood}}{\text{predictor prior probability}}$$

$p(\mathbf{x} \mid C)$ Likelihood
The probability of the document given the class

Rough Idea

- The value of a particular feature is independent of the value of any other feature, given the class variable
- All features contribute the same to the classification

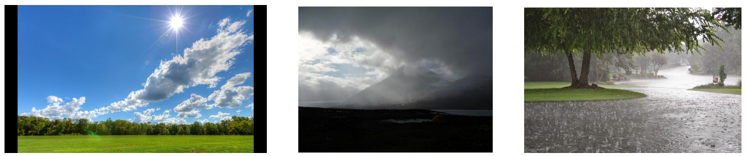
- Naïve Bayes' tries to find keywords in a set of documents that are predictive of the target (output) variable
- The internal coefficients will try to map tokens to scores
- Same as VADER, but without manually-created rules
  the machine will *estimate* them!

From (Lane et al., 2019, p. 65–68)

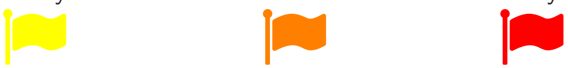
Naïve Bayes

A toy example: Should I ride my bike today?

One single factor: *zone (flag)*



sunny overcast rainy



(here come some dense slides)

Naïve Bayes

A toy example: Should I ride my bike today?

Dataset

Flag	
	yes
	yes
	no
	yes
	yes
	yes
	yes
	yes
	yes
	no
	no
	yes
	no
	no

Computing all the probabilities by "counting"

Frequency table

		
Flag	**yes**	**no**
	3	2
	4	0
	2	3

Likelihood table

		
Flag	**yes**	**no**
	3/9	2/5
	4/9	0/5
	2/9	3/5

Adapted from http://www.saedsayad.com/naive_bayesian.htm

Naïve Bayes

A toy example: Should I ride my bike today?

Likelihood table

Flag	yes	no
	3/9 [1]	2/5
	4/9	0/5
	2/9	3/5
	9/14 [2]	5/14

[1] $p(x \mid c) = p(\mid \text{yes}) = 3/9 = 0.33$

[2] $p(c) = p(\text{yes}) = 9/14 = 0.64$

$p(x) = p() = 5/14 = 0.36$

What is the Naïve Bayes' probability of yes if ?

$$
\begin{aligned}
p(c \mid x) &= p(c)p(x \mid c)/p(x) \\
p(\text{yes} \mid) &= p(\text{yes})p(\mid \text{yes})/p() \\
p(\text{yes} \mid) &= 0.64 * 0.33/0.36 \\
p(\text{yes} \mid) &= 0.59
\end{aligned}
$$

Adapted from http://www.saedsayad.com/naive_bayesian.htm

Naïve Bayes

A toy example: Should I ride my bike today?

If. . . let's ride !

Naïve Bayes

A toy example: Should I ride my bike today?

Considering more data

Flag	Temp	Humidity	Windy	
	hot	high	false	no
	hot	high	true	no
	hot	high	false	yes
	mild	high	false	yes
	cool	normal	false	yes
	cool	normal	true	no
	cool	normal	true	yes
	mild	high	false	no
	cool	normal	false	yes
	mild	normal	false	yes
	mild	normal	true	yes
	mild	high	true	yes
	hot	normal	false	yes
	mild	high		

Adapted from http://www.saedsayad.com/naive_bayesian.htm

Naïve Bayes

A toy example: Should I ride my bike today?

Frequency tables

Flag	yes	no
	3	2
	4	0
	2	3

Humidity	yes	no
high	3	4
normal	6	1

Temp	yes	no
hot	2	2
mild	4	2
cool	3	1

Windy	yes	no
false	6	2

Likelihood tables

Flag	yes	no
	3/9	2/5
	4/9	0/5
	2/9	3/5

Humidity	yes	no
high	3/9	4/5
normal	6/9	1/5

Temp	yes	no
hot	2/9	2/5
mild	4/9	2/5
cool	3/9	1/5

Windy	yes	no
false	6/9	2/5

Adapted from http://www.saedsayad.com/naive_bayesian.htm

Naïve Bayes

Likelihood tables

Flag	yes	no
	3/9	2/5
	4/9	0/5
	2/9	3/5

Temp	yes	no
hot	2/9	2/5
mild	4/9	2/5
cool	3/9	1/5

Humidity	yes	no
high	3/9	4/5
normal	6/9	1/5

Windy	yes	no
false	6/9	2/5
true	3/9	3/5

flag	temp	humidity	windy	ride
	cool	high	true	?

$$
\begin{aligned}
p(\text{yes} \mid x) &= \frac{p(\text{yes})p(\mid \text{yes})p(\text{cool} \mid \text{yes})p(\text{high} \mid \text{yes})p(\text{true} \mid \text{yes})}{p()p(\text{cool})p(\text{high})p(\text{true})} \\
&= \frac{9/14 \times 2/9 \times 3/9 \times 3/9 \times 3/9}{5/14 \times 4/14 \times 7/14 \times 6/14}
\end{aligned}
$$

Adapted from `http://www.saedsayad.com/naive_bayesian.htm`

Naïve Bayes

Back to the definition...

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{8}$$

The probability $p(\mathbf{x})$ is constant for any given input

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{\cancel{p(\mathbf{x})}} \tag{9}$$

$$p(c \mid \mathbf{x}) \propto p(c)p(\mathbf{x} \mid c) \tag{10}$$

Naïve Bayes

Back to the definition...

$$p(c \mid \mathbf{x}) \propto p(c)p(\mathbf{x} \mid c) \tag{11}$$

Remember that $\mathbf{x}$ is a vector

$$p(c \mid x_1 \ldots x_n) \propto p(c)p(x_1 \mid c) \times p(x_2 \mid c) \times \cdots \times p(x_n \mid c) \tag{12}$$

Eq. (12) can be rewritten as

$$p(c \mid x_1 \ldots x_n) \propto p(c) \prod_{i=1}^{n} p(x_i \mid c) \tag{13}$$

Naïve Bayes

The classification process

Back to the toy example

$$
\begin{aligned}
p(\text{yes} \mid x) &\propto p(\text{yes})p(\mid \text{yes})p(\text{cool} \mid \text{yes})p(\text{high} \mid \text{yes})p(\text{true} \mid \text{yes}) \\
&\propto 9/14 \times 2/9 \times 3/9 \times 3/9 \times 3/9 \\
&\propto 0.00529, \text{ which is \textbf{not} a probability}
\end{aligned}
$$

Classification: the maximum for all the classes

$$c \propto \arg\max_{c} p(c) \prod_{i=1}^{n} p(x_i \mid c) \tag{14}$$

```
compute p(yes|x)
compute p(no|x)
if p(yes|x) > p(no|x):
    yes
else:
    no
```

Naïve Bayes

Classification process

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{15}$$

The probability $p(\mathbf{x})$ is constant for any given input!

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{\cancel{p(\mathbf{x})}} \tag{16}$$

Back to the toy example, using Eq. (16)...

$$\begin{aligned} p(\text{yes} \mid x) &= p(\text{yes})p(\text{rainy} \mid \text{yes})p(\text{cool} \mid \text{yes})p(\text{high} \mid \text{yes})p(\text{true} \mid \text{yes}) \\ &= 9/14 \times 2/9 \times 3/9 \times 3/9 \times 3/9 \\ &= 0.00529 \text{not a probability!} \end{aligned}$$

Training a Machine Learning Model

The dataset

We need a bunch of items (documents) with their associated class

kind	examples
binary	{positive, negative}
	{0, 1}
	{-1, 1}
multiclass	{positive, neutral, negative}
	{0,1,2}

In our case, we need the sentiment:

d_1	pos	d_5	neg	d_9	neu
d_2	neu	d_6	neg	d_{10}	pos
d_3	pos	d_7	neg	d_{11}	neu
d_4	pos	d_8	pos	d_{12}	neg

The dataset

Option 1 Use a corpus created by somebody else

Option 2 Build your own corpus[3]

(a) You have/hire experts to do it
(b) You engage non-experts through gamification
(c) You hire non-experts through explicit crowdsourcing
(d) There are many other ways to get annotated data

[3] A lesson about this is going on now.

Let us go and build a classifier with a corpus built by Hutto and Gilbert (2014)[4]

If you are following NLP in Action, they instruct you to download and install their software companion:

`https://github.com/totalgood/nlpia`

[4] http://comp.social.gatech.edu/papers/icwsm14.vader.hutto.pdf

What I did on OsX and GNU Linux

I use pipenv[5]

```
$ pipenv install --skip-lock nlpia
```

On Github they explain how to install it with conda or pip if you plan to contribute to the project

</> Let us see it working

[5] `https://pipenv.readthedocs.io/en/latest/`

References

Hutto, C. and E. Gilbert
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